Buell Part A Solutions

EQUATIONS IN QUADRATIC FORM

A. Solve each of the following equations. Find ALL solutions, both real and complex.

1. $x^4 - 5x^2 + 4 = 0$
2. $x^4 - 10x^2 + 25 = 0$
3. $5 = 7x^2 - 2x^4$
4. $3 = 8x^2 - 4x^4$
5. $3x^4 - 2x^2 - 1 = 0$
6. $2x^4 - 5x^2 - 12 = 0$
7. $x^4 + 2x^2 - 15 = 0$
8. $3x^4 + 10x^2 - 25 = 0$
9. $2x^4 + x^2 - 2 = 0$
10. $x^4 - 8x^2 + 4 = 0$
11. $x^6 + 7x^3 - 8 = 0$
12. $x^6 - 7x^3 - 8 = 0$
13. $4x^{-2} - 9x^{-1} - 5 = 0$
14. $2x^{-2} - 3x^{-1} - 4 = 0$
15. $7x^{-2} + 19x^{-1} = 6$
16. $5x^{-2} - 43x^{-1} = 18$

B. Solve each of the following equations using substitution. Find ALL solutions, both real and complex.

1. $2(7x + 5)^2 - 5(7x + 5) = 3$
2. $3(1 - 3x)^2 + 5(1 - 3x) + 2 = 0$
3. $(2x + 7)^2 + 2(2x + 7) - 1 = 0$
4. $2(5x - 1)^2 + 2(5x - 1) + 3 = 0$
5. $\dfrac{1}{(3x+5)^2} = \dfrac{1}{3x+5} + 2$
6. $\dfrac{7}{2x-3} + \dfrac{3}{(2x-3)^2} = 6$
7. $8(5x - 4)^4 - 10(5x - 4)^2 + 3 = 0$
8. $6(3x + 2)^4 - 11(3x + 2)^2 + 4 = 0$
9. $(x^2 - 1)^2 + (x^2 - 1) - 12 = 0$
10. $(3x^2 + 7)^2 - 2(3x^2 + 7) - 15 = 0$
11. $\dfrac{17}{x^2+1} - \dfrac{22}{(x^2+1)^2} = 3$
12. $1 + 3(x^2 - 1)^{-1} = 28(x^2 - 1)^{-2}$
13. $3 + \dfrac{5}{x^2+1} = \dfrac{2}{(x^2+1)^2}$
14. $5(x^2 + 1)^{-2} = 4(x^2 + 1)^{-1} + 1$
15. $\left(\dfrac{x}{x+1}\right)^2 + \dfrac{2x}{x+1} = 8$
16. $\left(\dfrac{x}{x-1}\right)^2 = 6\left(\dfrac{x}{x-1}\right) + 7$
17. $x + \sqrt{x} = 20$
18. $x + x^{\frac{1}{2}} = 6$
19. $x + 8\sqrt{x} = 0$
20. $x - 8x^{\frac{1}{2}} = 0$
21. $\sqrt{x} - 2\sqrt[4]{x} + 1 = 0$
22. $x^{\frac{1}{2}} - 4x^{\frac{1}{4}} + 4 = 0$
23. $4\sqrt{x} - 9\sqrt[4]{x} + 4 = 0$
24. $x^{\frac{1}{2}} - 3x^{\frac{1}{4}} + 2 = 0$
25. $2\sqrt[3]{x^2} - 5\sqrt[3]{x} - 3 = 0$
26. $3x^{\frac{4}{3}} + 5x^{\frac{2}{3}} - 2 = 0$
27. $\sqrt[3]{(5x-1)^2} + \sqrt[3]{5x-1} = 12$
28. $(2x+3)^{\frac{2}{3}} - 2(2x+3)^{\frac{1}{3}} - 3 = 0$
29. $20\left(2-\sqrt{x}\right)^2 + 11\left(2-\sqrt{x}\right) = 3$
30. $2\left(1+2\sqrt{x}\right)^2 - \left(1+2\sqrt{x}\right) = 21$

Equations in Quadratic Form Part A

① $x^4 - 5x^2 + 4 = 0$

$(x^2-4)(x^2+1) = 0$

$x^2 = 4$ | $x^2 = +1$

$x = \pm 2$ | $x = \pm 1$ ✓

② $x^4 - 10x^2 + 25 = 0$

$(x^2-5)^2 = 0$

$x^2 - 5 = 0$

$x^2 = 5$

$x = \pm\sqrt{5}$ ✓

③ $5 = 7x^2 - 2x^4$

$2x^4 - 7x^2 + 5 = 0$

$(2x^2-5)(x^2-1) = 0$

$2x^2 = 5$ | $x^2 = 1$

$x^2 = \frac{5}{2}$ | $x = \pm 1$ ✓

$x = \pm\sqrt{\frac{5}{2}}$

$x = \pm\frac{\sqrt{10}}{2}$ ✓

④** $3 = 8x^2 - 4x^4$

$4x^4 - 8x^2 + 3 = 0$

$(2x^2-1)(2x^2-3) = 0$

$2x^2 = 1$ | $2x^2 = 3$

$x^2 = \frac{1}{2}$ | $x^2 = \frac{3}{2}$

$x = \pm\sqrt{\frac{1}{2}}$ | $x = \pm\sqrt{\frac{3}{2}}$

$x = \pm\frac{\sqrt{2}}{2}$ ✓ | $x = \pm\frac{\sqrt{6}}{2}$ ✓

⑤ $3x^4 - 2x^2 - 1 = 0$

$(3x^2+1)(x^2-1) = 0$

$3x^2 = -1$ | $x^2 = 1$ ✓

$x^2 = -\frac{1}{3}$ | $x = \pm 1$

$x = \pm\frac{i}{\sqrt{3}}$ ✓

$x = \pm\frac{i\sqrt{3}}{3}$

⑥ $2x^4 - 5x^2 - 12 = 0$

$(2x^2+3)(x^2-4) = 0$

$2x^2 = -3$ | $x^2 = 4$

$x^2 = -\frac{3}{2}$ | $x = \pm 2$ ✓

$x = \pm\frac{i\sqrt{3}}{\sqrt{2}}$ ✓

$x = \pm\frac{i\sqrt{6}}{2}$

⑦ $x^4 + 2x^2 - 15 = 0$

$(x^2+5)(x^2-3) = 0$

$x^2 = -5$ | $x^2 = 3$ ✓

$x = \pm i\sqrt{5}$ | $x = \pm\sqrt{3}$

⑧ $3x^4 + 10x^2 - 25 = 0$

$(3x^2-5)(x^2+5) = 0$

$3x^2 - 5$ | $x^2 = -5$ ✓

$x^2 = \frac{5}{3}$ | $x = \pm i\sqrt{5}$

$x = \pm\frac{\sqrt{5}}{\sqrt{3}}$ ✓

$x = \pm\frac{\sqrt{15}}{3}$

⑨ $2x^4 + x^2 - 2 = 0$

$a = 2 \quad b = 1 \quad c = -2$

$x^2 = \frac{-1 \pm \sqrt{1 - 4(2)(-2)}}{4}$

$x^2 = \frac{-1 \pm \sqrt{17}}{4}$

$x = \pm\sqrt{\frac{-1 \pm \sqrt{17}}{4}}$ ✓

$\left\{\pm\frac{\sqrt{-1+\sqrt{17}}}{2}, \frac{\sqrt{-1-\sqrt{17}}}{2}, -\frac{\sqrt{-1+\sqrt{17}}}{2}, \frac{\sqrt{-1-\sqrt{17}}}{2}\right\}$ ✓

⑩ $x^4 - 8x^2 + 4 = 0$

$a = 1 \quad b = -8 \quad c = 4$

$x^2 = \frac{8 \pm \sqrt{64 - 4(1)(4)}}{2}$

$x^2 = \frac{8 \pm \sqrt{48}}{2}$

$x^2 = \frac{8 \pm 4\sqrt{3}}{2}$

$x^2 = 4 \pm 2\sqrt{3}$

$x = \pm\sqrt{4 \pm 2\sqrt{3}}$

$\left\{\sqrt{4+2\sqrt{3}}, -\sqrt{4+2\sqrt{3}}, \sqrt{4-2\sqrt{3}}, -\sqrt{4+2\sqrt{3}}\right\}$ ✓

Eg. in Quad Form

Part A Continued

(11) $x^6+7x^3-8=0$

$(x^3+8)(x^3-1)=0$

$(x+2)(x^2-2x+4)(x-1)(x^2+x+1)=0$

$x=-2$ | $a=1\ b=-2\ c=4$ | $x=1$ | $a=1\ b=1\ c=1$

$x=\frac{+2\pm\sqrt{4-4(1)(4)}}{2}$ $x=\frac{-1\pm\sqrt{1-4(1)(1)}}{2}$

$x=\frac{+2\pm\sqrt{-12}}{2}$ $x=\frac{-1\pm\sqrt{-3}}{2}$

$x=\frac{+2\pm 2i\sqrt{3}}{2}$ $x=\frac{-1\pm i\sqrt{3}}{2}$

$x=1\pm i\sqrt{3}$ $x=-\frac{1}{2}\pm\frac{i\sqrt{3}}{2}$

$\left\{-2,\ 1,\ 1\pm i\sqrt{3},\ -\frac{1}{2}\pm\frac{i\sqrt{3}}{2}\right\}$ ✓

(12) $x^6-7x^3-8=0$

$(x^3-8)(x^3+1)=0$

$x^3-8=0$ $x^3+1=0$

$(x-2)(x^2+2x+4)=0$ $(x+1)(x^2-x+1)=0$

$x=2$ $a=1\ b=2\ c=4$ $x=-1$ $a=1\ b=-1\ c=1$

$x=\frac{-2\pm\sqrt{4-4(1)(4)}}{2}$ $x=\frac{1\pm\sqrt{1-4(1)(1)}}{2}$

$x=\frac{-2\pm\sqrt{-12}}{2}$ $x=\frac{1\pm\sqrt{-3}}{2}$

$x=\frac{-2\pm 2i\sqrt{3}}{2}$ $x=\frac{1}{2}\pm\frac{i\sqrt{3}}{2}$

$x=-1\pm i\sqrt{3}$

$\left\{2,\ -1,\ -1\pm i\sqrt{3},\ \frac{1}{2}\pm\frac{i\sqrt{3}}{2}\right\}$

(13) $4x^{-2}-9x^{-1}-5=0$

$\left(\frac{4}{x^2}-\frac{9}{x}-5=0\right)x^2$

$4-9x-5x^2=0$

$5x^2+9x-4=0$

$x=\frac{-9\pm\sqrt{81-4(5)(-4)}}{10}$

$x=\frac{-9\pm\sqrt{81+80}}{10}$

$x=\frac{-9\pm\sqrt{161}}{10}$

(14) $2x^{-2}-3x^{-1}-4=0$

$\left(\frac{2}{x^2}-\frac{3}{x}-4=0\right)x^2$

$2-3x-4x^2=0$

$4x^2+3x-2=0$

$x=\frac{-3\pm\sqrt{9-4(4)(-2)}}{8}$

$x=\frac{-3\pm\sqrt{9+32}}{8}$

$x=\frac{-3\pm\sqrt{41}}{8}$ ✓

(15) $7x^{-2} + 19x^{-1} = 6$

$\left(\frac{7}{x^2} + \frac{19}{x} = 6\right)x^2$

$7 + 19x = 6x^2$

$0 = 6x^2 - 19x - 7$

$(3x + 1)(2x - 7) = 0$

$x = -\frac{1}{3} \quad x = \frac{7}{2}$

$\left\{-\frac{1}{3}, \frac{7}{2}\right\}$ ✓

(16) $\left(5x^{-2} - 43x^{-1} = 18\right)x^2$

$5 - 43x = 18x^2$

$18x^2 + 43x - 5 = 0$

$(9x - 1)(2x + 5) = 0$

$x = \frac{1}{9} \quad x = \frac{-5}{2}$

$\left\{\frac{1}{9}, \frac{-5}{2}\right\}$ ✓

Part B Solutions

EQUATIONS IN QUADRATIC FORM

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5. $3x^4 - 2x^2 - 1 = 0$
6. $2x^4 - 5x^2 - 12 = 0$
7. $x^4 + 2x^2 - 15 = 0$
8. $3x^4 + 10x^2 - 25 = 0$
9. $2x^4 + x^2 - 2 = 0$
10. $x^4 - 8x^2 + 4 = 0$
11. $x^6 + 7x^3 - 8 = 0$
12. $x^6 - 7x^3 - 8 = 0$
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B. Solve each of the following equations using substitution. Find ALL solutions, both real and complex.

1. $2(7x + 5)^2 - 5(7x + 5) = 3$
2. $3(1 - 3x)^2 + 5(1 - 3x) + 2 = 0$
3. $(2x + 7)^2 + 2(2x + 7) - 1 = 0$
4. $2(5x - 1)^2 + 2(5x - 1) + 3 = 0$
5. $\dfrac{1}{(3x+5)^2} = \dfrac{1}{3x+5} + 2$
6. $\dfrac{7}{2x-3} + \dfrac{3}{(2x-3)^2} = 6$
7. $8(5x - 4)^4 - 10(5x - 4)^2 + 3 = 0$
8. $6(3x + 2)^4 - 11(3x + 2)^2 + 4 = 0$
9. $(x^2 - 1)^2 + (x^2 - 1) - 12 = 0$
10. $(3x^2 + 7)^2 - 2(3x^2 + 7) - 15 = 0$
11. $\dfrac{17}{x^2+1} - \dfrac{22}{(x^2+1)^2} = 3$
12. $1 + 3(x^2 - 1)^{-1} = 28(x^2 - 1)^{-2}$
13. $3 + \dfrac{5}{x^2+1} = \dfrac{2}{(x^2+1)^2}$
14. $5(x^2 + 1)^{-2} = 4(x^2 + 1)^{-1} + 1$
15. $\left(\dfrac{x}{x+1}\right)^2 + \dfrac{2x}{x+1} = 8$
16. $\left(\dfrac{x}{x-1}\right)^2 = 6\left(\dfrac{x}{x-1}\right) + 7$
17. $x + \sqrt{x} = 20$
18. $x + x^{\frac{1}{2}} = 6$
19. $x + 8\sqrt{x} = 0$
20. $x - 8x^{\frac{1}{2}} = 0$
21. $\sqrt{x} - 2\sqrt[4]{x} + 1 = 0$
22. $x^{\frac{1}{2}} - 4x^{\frac{1}{4}} + 4 = 0$
23. $4\sqrt{x} - 9\sqrt[4]{x} + 4 = 0$
24. $x^{\frac{1}{2}} - 3x^{\frac{1}{4}} + 2 = 0$
25. $2\sqrt[3]{x^2} - 5\sqrt[3]{x} - 3 = 0$
26. $3x^{\frac{4}{3}} + 5x^{\frac{2}{3}} - 2 = 0$
27. $\sqrt[3]{(5x-1)^2} + \sqrt[3]{5x-1} = 12$
28. $(2x+3)^{\frac{2}{3}} - 2(2x+3)^{\frac{1}{3}} - 3 = 0$
29. $20\left(2-\sqrt{x}\right)^2 + 11\left(2-\sqrt{x}\right) = 3$
30. $2\left(1+2\sqrt{x}\right)^2 - \left(1+2\sqrt{x}\right) = 21$

Equations in Quadratic Form B.

① $2(7x+5)^2 - 5(7x+5) = 3$ let $y = 7x+5$

$2y^2 - 5y - 3 = 0$

$(2y+1)(y-3) = 0$

$y = -\frac{1}{2}$ $\quad y = 3$

$7x+5 = -\frac{1}{2}$ $\quad 7x+5 = 3$

$7x = -\frac{11}{2}$ $\quad 7x = -2$

$x = -\frac{11}{14}$ $\quad x = -\frac{2}{7}$

$\left\{-\frac{11}{14}, -\frac{2}{7}\right\}$ ✓

② $3(1-3x)^2 + 5(1-3x) + 2 = 0$ let $y = 1-3x$

$3y^2 + 5y + 2 = 0$

$(3y+2)(y+1) = 0$

$3y+2 = 0$ | $y+1 = 0$

$y = -\frac{2}{3}$ $\quad y = -1$

$1-3x = -\frac{2}{3}$ $\quad 1-3x = -1$

$-3x = -\frac{5}{3}$ $\quad -3x = -2$

$x = \frac{5}{9}$ $\quad x = \frac{2}{3}$

$\left\{\frac{5}{9}, \frac{2}{3}\right\}$ ✓

③ $(2x+7)^2 + 2(2x+7) - 1 = 0$ let $y = 2x+7$

$y^2 + 2y - 1 = 0$

$a=1 \quad b=2 \quad c=-1$

$y = \frac{-2 \pm \sqrt{4-4(1)(-1)}}{2} = \frac{-2 \pm \sqrt{8}}{2}$

$y = -1 \pm \sqrt{2}$

$2x+7 = -1 \pm \sqrt{2}$

$2x = -8 \pm \sqrt{2}$

$x = \frac{-8 \pm \sqrt{2}}{2}$

$\left\{\frac{-8+\sqrt{2}}{2}, \frac{-8-\sqrt{2}}{2}\right\}$ ✓

④ $2(5x-1)^2 + 2(5x-1) + 3 = 0$ let $y = 5x-1$

$2y^2 + 2y + 3 = 0$

$a=2 \quad b=2 \quad c=3$

$y = \frac{-2 \pm \sqrt{4-4(2)(3)}}{4} = \frac{-2 \pm \sqrt{-20}}{4} = \frac{-1 \pm i\sqrt{5}}{2}$

$5x-1 = \frac{-1 \pm i\sqrt{5}}{2}$

$5x = \frac{-1 \pm i\sqrt{5}}{2}$

$x = \frac{-1 \pm i\sqrt{5}}{2}$

$\left\{-\frac{1}{2} + \frac{i\sqrt{5}}{2}, -\frac{1}{2} - \frac{i\sqrt{5}}{2}\right\}$ ✓

⑤ $\frac{1}{(3x+5)^2} = \frac{1}{3x+5} + 2$ let $y = 3x+5$

$y^2\left[\frac{1}{y^2} = \frac{1}{y} + 2\right]$

$1 = y + 2y^2$

$0 = 2y^2 + y - 1$

$0 = (2y-1)(y+1)$

$y = \frac{1}{2}$ $\quad y = -1$

$3x+5 = \frac{1}{2}$ $\quad 3x+5 = -1$

$3x = -\frac{9}{2}$ $\quad 3x = -6$

$x = -\frac{9}{6} = -\frac{3}{2}$ $\quad x = -2$

$\{-3/2, -2\}$ ✓

⑥ $\frac{7}{2x-3} + \frac{3}{(2x-3)^2} = 6$ let $y = 2x-3$

$\left[\frac{7}{y} + \frac{3}{y^2} = 6\right]y^2$

$7y + 3 = 6y^2$

$0 = 6y^2 - 7y - 3$

$0 = (3y+1)(2y-3)$

$y = -1/3$ | $y = \frac{3}{2}$

$2x-3 = -\frac{1}{3}$ $\quad 2x-3 = \frac{3}{2}$

$2x = \frac{8}{3}$ $\quad 2x = \frac{9}{2}$

$x = \frac{8}{6} = \frac{4}{3}$ $\quad x = \frac{9}{4}$

$\left\{\frac{9}{4}, \frac{4}{3}\right\}$ ✓

⑦ $8(5x-4)^4 - 10(5x-4)^2 + 3 = 0$ let $y = (5x-4)^2$

$8y^2 - 10y + 3 = 0$

$(4y - 3)(2y - 1) = 0$

$y = \frac{3}{4}$ | $y = \frac{1}{2}$

$(5x-4)^2 = \frac{3}{4}$

$5x - 4 = \pm\frac{\sqrt{3}}{2}$

$5x = \frac{8 \pm \sqrt{3}}{2}$

$x = \frac{8 \pm \sqrt{3}}{10}$ ✓

$(5x-4)^2 = \frac{1}{2}$

$5x - 4 = \pm\frac{\sqrt{2}}{2}$

$5x = \frac{8 \pm \sqrt{2}}{2}$

$x = \frac{8 \pm \sqrt{2}}{10}$ ✓

⑧ $6(3x+2)^4 - 11(3x+2)^2 + 4 = 0$ $y = (3x+2)^2$

$6y^2 - 11y + 4 = 0$

$(3y - 4)(2y - 1) = 0$

$y = \frac{4}{3}$ | $y = \frac{1}{2}$

$(3x+2)^2 = \frac{4}{3}$

$3x + 2 = \pm\frac{2}{\sqrt{3}}$

$3x = -2 \pm \frac{2\sqrt{3}}{3}$

$3x = \frac{-6 \pm 2\sqrt{3}}{3}$

$x = \frac{-6 \pm 2\sqrt{3}}{9}$ ✓

$(3x+2)^2 = \frac{1}{2}$

$3x + 2 = \pm\frac{\sqrt{2}}{2}$

$3x = \frac{4 \pm \sqrt{2}}{2}$

$x = \frac{4 \pm \sqrt{2}}{6}$ ✓

⑨ $(x^2-1)^2 + (x^2-1) - 12 = 0$ let $y = (x^2-1)$

$y^2 + y - 12 = 0$

$(y + 4)(y - 3) = 0$

$y = -4$ | $y = 3$

$x^2 - 1 = -4$ | $x^2 - 1 = 3$

$x^2 = -3$ | $x^2 = 4$

$x = \pm i\sqrt{3}$ ✓ | $x = \pm 2$ ✓

⑩ $(3x^2+7)^2 - 2(3x^2+7) - 15 = 0$ let $y = 3x^2+7$

$y^2 - 2y - 15 = 0$

$(y - 5)(y + 3) = 0$

$y = 5$ | $y = -3$

$3x^2 + 7 = 5$ | $3x^2 + 7 = -3$

$3x^2 = -2$ | $3x^2 = -10$

$x^2 = -\frac{2}{3}$ | $x^2 = \frac{-10}{3}$

$x = \pm i\sqrt{\frac{2}{3}}$ | $x = \pm\sqrt{\frac{10}{3}}$

$x = \pm\frac{i\sqrt{6}}{3}$ ✓ | $x = \pm\frac{\sqrt{30}}{3}$ ✓

⑪ $\frac{17}{x^2+1} - \frac{22}{(x^2+1)^2} = 3$ let $y = (x^2+1)$

$\frac{17}{y} - \frac{22}{y^2} = 3$

$17y - 22 = 3y^2$

$0 = 3y^2 - 17y + 22$

$0 = (y - 2)(3y - 11)$

$y = 2$ | $y = \frac{11}{3}$

$x^2 + 1 = 2$ | $x^2 + 1 = \frac{11}{3}$

$x^2 = 1$ | $x^2 = \frac{8}{3}$

$x = \pm 1$ ✓ | $x = \pm\frac{2\sqrt{2}}{\sqrt{3}}$

$x = \pm\frac{2\sqrt{6}}{3}$ ✓

⑫ $1 + 3(x^2-1)^{-1} = 28(x^2-1)^{-2}$ let $y = (x^2-1)^{-1}$

$1 + 3y = 28y^2$

$0 = 28y^2 - 3y - 1$

$0 = (7y + 1)(4y - 1)$

$y = -\frac{1}{7}$ | $y = \frac{1}{4}$

$(x^2-1)^{-1} = -\frac{1}{7}$ | $(x^2-1)^{-1} = \frac{1}{4}$

$x^2 - 1 = -7$ | $x^2 - 1 = 4$

$x^2 = -6$ | $x^2 = 5$

$x = \pm i\sqrt{6}$ ✓ | $x = \pm\sqrt{5}$ ✓

Eqs in Quad form Part B continued

(13) let $y = x^2+1$

$3 + \frac{5}{x^2+1} = \frac{2}{(x^2+1)^2}$

$3 + \frac{5}{y} = \frac{2}{y^2}$

$3y^2 + 5y = 2$

$3y^2 + 5y - 2 = 0$

$(3y-1)(y+2) = 0$

$y = \frac{1}{3} \mid y = -2$

$x^2+1 = \frac{1}{3} \quad x^2+1 = -2$

$x^2 = -\frac{2}{3} \quad x^2 = -3$

$x = \pm\frac{i\sqrt{6}}{3} \quad x = \pm i\sqrt{3}$ ✓

(14) let $y = (x^2+1)^{-1}$

$5(x^2+1)^{-2} = 4(x^2+1)^{-1} + 1$

$5y^2 = 4y + 1$

$5y^2 - 4y - 1 = 0$

$(5y+1)(y-1) = 0$

$y = -\frac{1}{5} \mid y = 1$

$(x^2+1)^{-1} = -\frac{1}{5} \quad x^2+1 = 1$

$x^2 = 0$

$x^2+1 = -5$ $\quad x = 0$ ✓

$x^2 = -6$ ✓

$x = \pm i\sqrt{6}$

(15) let $y = \frac{x}{x+1}$

$\left(\frac{x}{x+1}\right)^2 + \left(\frac{2x}{x+1}\right) = 8$

$y^2 + 2y = 8$

$y^2 + 2y - 8 = 0$

$(y-2)(y+4) = 0$

$y = 2 \mid y = -4$

$\frac{x}{x+1} = 2 \quad \frac{x}{x+1} = -4$

$x = 2x + 2 \quad x = -4x - 4$

$-2 = x$ ✓ $\quad -4 = 5x$

$x = -\frac{4}{5}$ ✓

(16) let $y = \frac{x}{x-1}$

$\left(\frac{x}{x-1}\right)^2 = 6\left(\frac{x}{x-1}\right) + 7$

$y^2 = 6y + 7$

$y^2 - 6y - 7 = 0$

$(y+1)(y-7) = 0$

$y = -1 \quad y = 7$

$\frac{x}{x-1} = -1 \quad \frac{x}{x-1} = 7$

$x = -x + 1 \quad x = 7x - 7$

$2x = 1 \quad -6x = -7$

$x = \frac{1}{2} \quad x = \frac{7}{6}$

$\left\{\frac{1}{2}, \frac{7}{6}\right\}$ ✓

(17) let $y = \sqrt{x}$

$x + \sqrt{x} = 20$

$y^2 + y = 20$

$y^2 + y - 20 = 0$

$(y+5)(y-4) = 0$

$y = -5 \mid y = 4$

$\sqrt{x} \neq -5 \quad \sqrt{x} = 4$ ✓

$x = 16$

(18) let $y = x^{1/2}$

$x + x^{1/2} = 6$

$y^2 + y = 6$

$y^2 + y - 6 = 0$

$(y+3)(y-2) = 0$

$y = -3 \mid y = 2$

$x^{1/2} = -3 \quad x^{1/2} = 2$

$x = 4$ ✓

(19) let $y = \sqrt{x}$

$x + 8\sqrt{x} = 0$

$y^2 + 8y = 0$

$y(y+8) = 0$

$y = 0, \; y = -8$

$\sqrt{x} = 0 \quad \sqrt{x} \neq -8$

$x = 0$ ✓

(20) let $y = \sqrt{x}$

$x - 8x^{1/2} = 0$

$x - 8\sqrt{x} = 0$

$y^2 - 8y = 0$

$y(y-8) = 0$

$y = 0, \; y = 8$

$\sqrt{x} = 0 \quad \sqrt{x} = 8$

$x = 0 \quad x = 64$ ✓

$\{0, 64\}$

(21) let $y = \sqrt[4]{x}$

$\sqrt{x} - 2\sqrt[4]{x} + 1 = 0$

$y^2 - 2y + 1 = 0$ ✓

$(y-1)^2 = 0$

$y = 1$

$\sqrt[4]{x} = 1$ $\quad x = 1$

$y = \sqrt[4]{x}$

$y = \frac{9 \pm \sqrt{81 - 4(4)(4)}}{8}$

$y = \frac{9 \pm \sqrt{17}}{8}$

$y = \frac{9 \pm \sqrt{17}}{8}$

$\sqrt[4]{x} = \frac{9 \pm \sqrt{17}}{8}$

$x = \frac{(9 \pm \sqrt{17})^4}{512}$ ✓

(22) let $y = x^{1/4}$

$x^{1/2} - 4x^{1/4} + 4 = 0$

$y^2 - 4y + 4 = 0$

$(y-2)^2 = 0$ ✓

$y = 2$ $\quad x = 16$

$\sqrt[4]{x} = 2$

(23) $4\sqrt{x} - 9\sqrt[4]{x} + 4 = 0$

$4y^2 - 9y + 4 = 0$

$a = 4 \; b = -9 \; c = 4$

Eq. in Quad form Part B - continued

(24) let $y = x^{1/4}$

$x^{\frac{1}{2}} - 3x^{\frac{1}{4}} + 2 = 0$

$y^2 - 3y + 2 = 0$

$(y-2)(y-1) = 0$

$y = 2 \quad y = 1$

$\sqrt[4]{x} = 2 \quad \sqrt[4]{x} = 1$

$x = 16 \quad x = 1$

$\{16, 1\}$ ✓

(25) $2\sqrt[3]{x^2} - 5\sqrt[3]{x} - 3 = 0$

let $y = \sqrt[3]{x}$

$2y^2 - 5y - 3 = 0$

$(2y+1)(y-3) = 0$

$y = -\frac{1}{2} \quad y = 3$

$\sqrt[3]{x} = -\frac{1}{2} \quad \sqrt[3]{x} = 3$

$x = -\frac{1}{8} \quad x = 27$ ✓

(26) let $y = x^{2/3}$

$3x^{4/3} + 5x^{2/3} - 2 = 0$

$3y^2 + 5y - 2 = 0$

$(3y-1)(y+2) = 0$

$y = \frac{1}{3} \quad y = -2$

$x^{2/3} = \frac{1}{3} \quad x^{2/3} = -2$

$\sqrt[3]{x^2} = \frac{1}{3} \quad \sqrt[3]{x^2} = -2$

$x^2 = \frac{1}{27} \quad x^2 = -8$

$x = \pm\frac{1}{2\sqrt{3}} \quad x = \pm 2i\sqrt{2}$

$x = \pm\frac{\sqrt{3}}{9}$ ✓

(27) $\sqrt[3]{(5x-1)^2} + \sqrt[3]{5x-1} = 12$

let $y = \sqrt[3]{5x-1}$

$y^2 + y = 12$

$y^2 + y - 12 = 0$

$(y-3)(y+4) = 0$

$y = 3 \quad y = -4$

$\sqrt[3]{5x-1} = 3 \quad \sqrt[3]{5x-1} = -4$

$5x - 1 = 27 \quad 5x - 1 = -64$

$5x = 28 \quad 5x = -63$

$x = \frac{28}{5} \quad x = -\frac{63}{5}$ ✓

(28) let $y = (2x+3)^{1/3}$

$(2x+3)^{2/3} - 2(2x+3)^{1/3} - 3 = 0$

$y^2 - 2y - 3 = 0$

$(y-3)(y+1) = 0$

$y = 3 \quad y = -1$

$\sqrt[3]{2x+3} = 3 \quad \sqrt[3]{2x+3} = -1$

$2x + 3 = 27 \quad 2x + 3 = -1$

$2x = 24 \quad 2x = -4$

$x = 12 \quad x = -2$ ✓

(29) $20(2-\sqrt{x})^2 + 11(2-\sqrt{x}) = 3$

let $y = 2 - \sqrt{x}$

$20y^2 + 11y = 3$

$20y^2 + 11y - 3 = 0$

$(5y-1)(4y+3) = 0$

$y = \frac{1}{5} \quad 4y = -3$

$y = \frac{-3}{4}$

$2 - \sqrt{x} = \frac{1}{5} \quad 2 - \sqrt{x} = -\frac{3}{4}$

$-\sqrt{x} = -\frac{9}{5} \quad -\sqrt{x} = -\frac{11}{4}$

$\sqrt{x} = \frac{9}{5} \quad \sqrt{x} = \frac{11}{4}$

$x = \frac{81}{25}$ ✓ $\quad x = \frac{121}{16}$ ✓

(30) $2(1+2\sqrt{x})^2 - (1+2\sqrt{x}) = 21 \quad \times 20$

let $y = 1 + 2\sqrt{x}$

$2y^2 - y = 21$

$2y^2 - y - 21 = 0$

$(2y-7)(y+3) = 0$

$y = \frac{7}{2} \quad y = -3$

$1 + 2\sqrt{x} = \frac{7}{2} \quad 1 + 2\sqrt{x} = -3$

$2\sqrt{x} = \frac{5}{2} \quad 2\sqrt{x} = -4$

$\sqrt{x} = \frac{5}{4} \quad \sqrt{x} \neq -2$

$x = \frac{25}{16}$ ✓